Introduction to *The Geometer's Sketchpad*® Dynamic Geometry Software

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This tutorial is intended to provide a general overview of some of the capabilities of *The Geometer's Sketchpad*® dynamic geometry software. Examples of the wide range of sketches available on this powerful tool are included in the following pages.

Use *The Geometer's Sketchpad*® to Construct a Working Model of the Pythagorean Theorem

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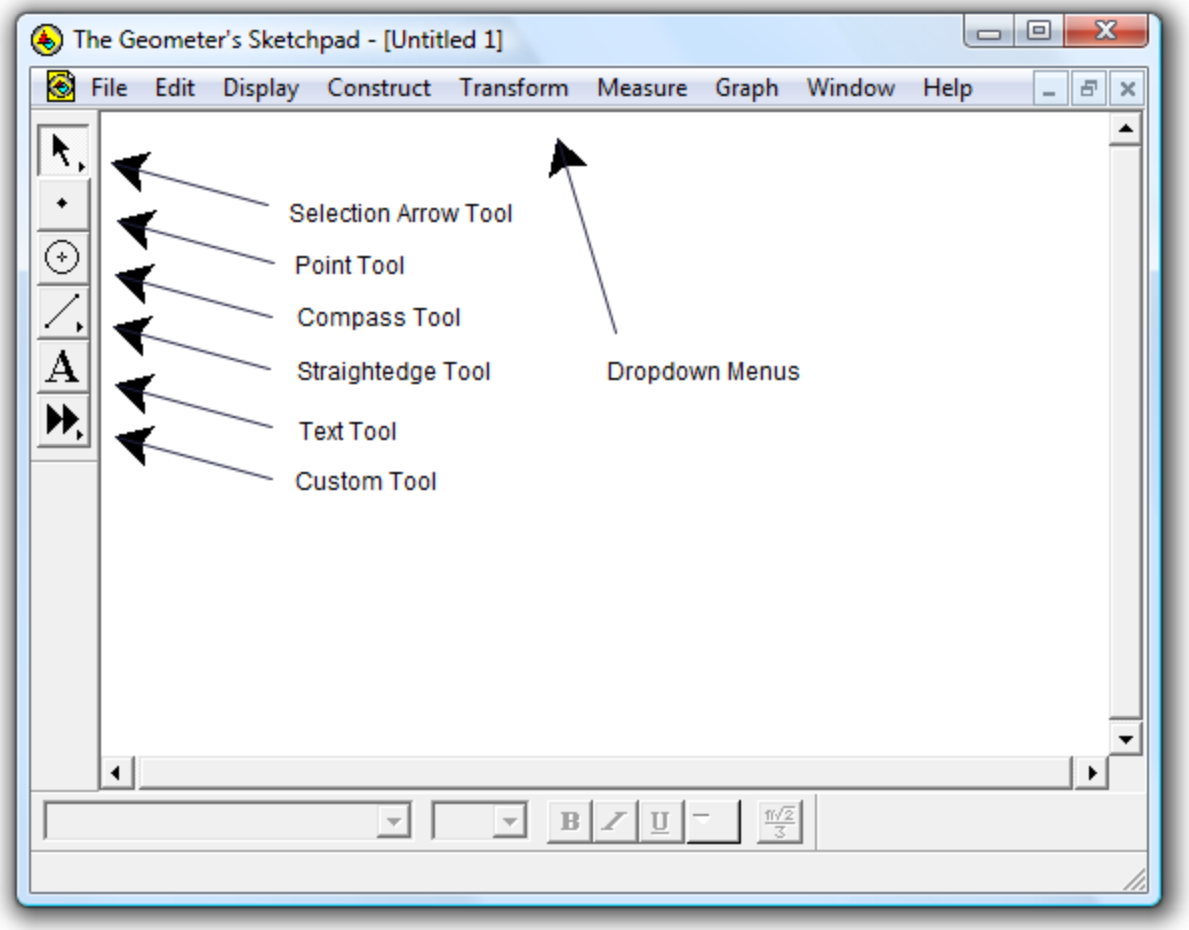
In this activity, you will learn how to use the basic tools and menus of *The Geometer's Sketchpad*® to construct a dynamic working model the Pythagorean Theorem.

What is *The Geometer's Sketchpad*®?

The Geometer's Sketchpad® (GSP) is dynamic geometry software useful for exploring plane geometry, algebra, calculus, statistics, and other branches of mathematics. It is possible to simulate three-dimensional geometry as well. Thousands of sketches are available on the Internet, most of them free. Version 4 is most commonly found in schools, although Version 5 was released late in 2009. Sketches are upwardly compatible. Version 5 also includes a provision for saving files in a format that will work on Version 4.

The Workspace

When you run GSP, the workspace looks like this:



The tools buttons are at the side, and dropdown menus are along the top.

Draw a Right Triangle:

- Select the **Straightedge Tool**.
- Click and drag to draw a line segment bounded by two points.

Tip: You can draw a horizontal, vertical, or diagonal segment by holding down the **Shift** key.

Next, fix an angle at 90°.

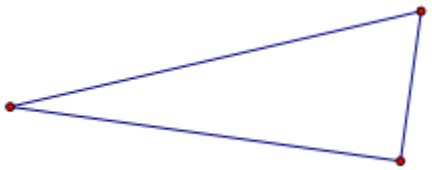
- Select the right endpoint of the segment.
- Select the segment.
- From the **Construct** menu, select **Perpendicular Line**.

A line perpendicular to the segment is drawn through the point.

- Select the **Point Tool**.
- Draw a point on the perpendicular line.
- Select the perpendicular line.
- From the **Display** menu, select **Hide Perpendicular Line**.

Tip: Alternatively, hold down the **CTRL** key, and press h.

- Use the **Straightedge Tool** to draw segments that complete the triangle.

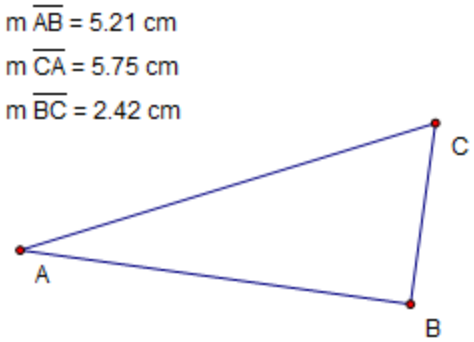


Click and drag each of the vertices in turn. Notice that the triangle remains a right triangle at all times.

Measure Lengths

- Select the three sides of the triangle.
- From the **Measure** menu, select **Length**.

Labels will appear for each of the vertices. Three length measurements will be displayed.



Save the Sketch

- From the **File** menu, select **Save As...** .

Navigate to a suitable directory, and select a descriptive name for your sketch.

- Select **Save**.

Calculate Squares

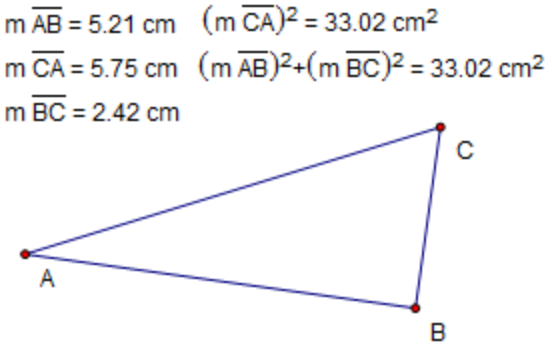
- From the **Measure** menu, select **Calculate...** .

The calculator box will appear**.**

- Click on the measure of the length of the hypotenuse.
- Click on ^, then 2 to complete the square.
- Click on **OK**.

The square of the hypotenuse will appear.

- From the **Measure** menu, select **Calculate...** .
- Click on the measure of one of the sides.
- Click on ^, then 2 to complete the square.
- Click on +.
- Click on the measure of the other side.
- Click on ^, then 2 to complete the square.
- Click on **OK**.



The sum of the squares of the other two sides will appear.

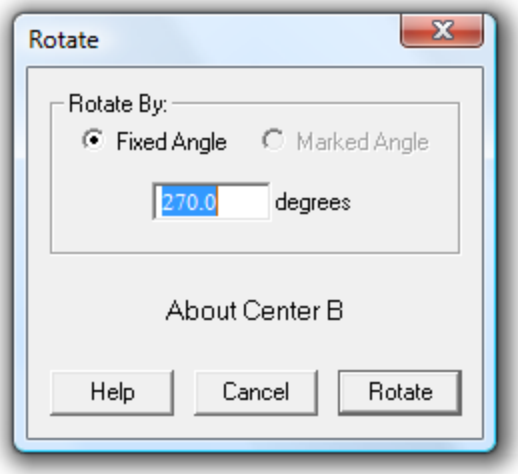
Tip: You can use the **Selection Arrow Tool** to move a measurement to a convenient location in the workspace.

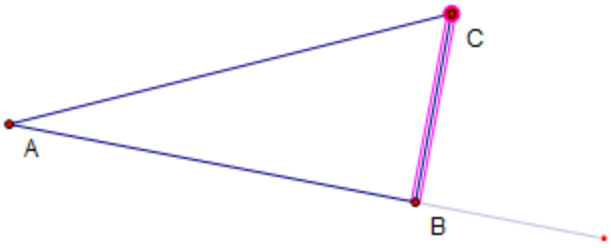
Click and drag each of the vertices in turn. Note that the Pythagorean relationship holds at all times.

Draw Squares on the Sides

To complete the model, you will draw squares on the sides of the triangle. These will change dynamically as you drag the vertices.

- Double-click on the vertex containing the right angle. This marks the point as the centre of rotation.
- Select one of the sides, and the vertex at the end of the side.
- From the **Transform** menu, select **Rotate...** .





A faint point will be plotted to show you the location of the rotated point and side. Note that

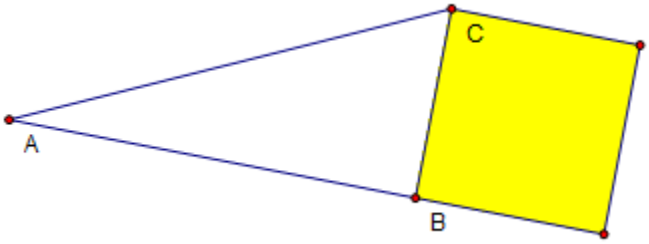
rotations are counter-clockwise.

- Change the angle to 270°.
- Click on **Rotate**.

The rotated segment and point will appear.

- Double-click on point C to mark it as the centre of rotation.
- Rotate B and the segment BC about C.
- Use the **Straightedge Tool** to complete the square.
- Select the four vertices of the square.
- From the **Construct** menu, select **Quadrilateral Interior**.

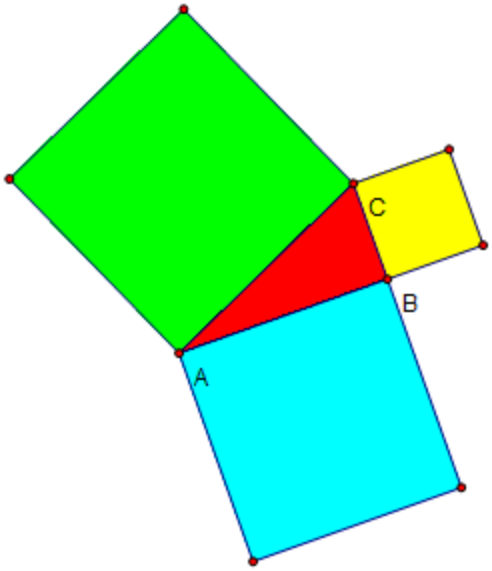
The square will be filled in with colour. You can select the interior, and change the colour by selecting **Color** from the **Display** menu.



Tip: Alternatively, you can right-click on the interior, and select **Color**.

Use this procedure to draw squares on the other sides of the triangle. Select colours of your choice.

Finally, construct a triangle interior for the right triangle, and colour it.



Save your sketch.

Add a Page

You can add pages to your sketch.

- From the **File** menu, select **Document Options...** .
- Press **Add Page**.
- Select **Blank Page**.
- Press **OK**.

Your new page will appear. You could also have chosen to duplicate the first page.

You can toggle among the pages using the buttons at the lower left of the workspace. You can also use **Document Options** to change the names of the pages.

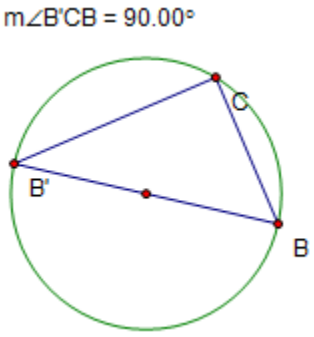
Another Way to Draw a Right Triangle

- Select the **Compass Tool**.
- Draw a circle in the workspace.

Note that the circle is defined by a centre, and a point on the circle.

- Use the **Transform** menu to rotate the point on the circle 180° about the centre.
- Use the **Straightedge Tool** to draw a diameter.
- Use the **Point Tool** to draw another point on the circle.
- Use the **Straightedge Tool** to complete a triangle in the semicircle.
- Select the vertices of the triangle such that the point in the semicircle is the second one selected.
- From the **Measure** menu, select **Angle**.

The measure of the angle in the semicircle will appear. Drag each of the vertices of the triangle in turn, and observe what happens.



- Hide the circle and its centre.

You can now complete a model of the Pythagorean Theorem using this triangle, if desired.

Use *The Geometer's Sketchpad*® to Create and Save Custom Tools

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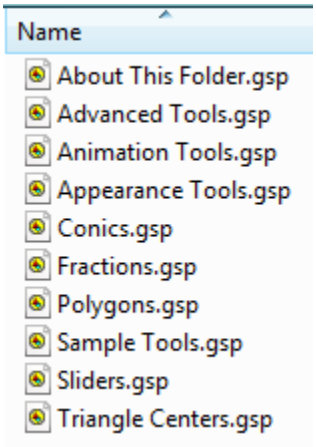
In this activity, you will learn how to use *The Geometer's Sketchpad*® to create custom tools, and save them such that they are available for all documents.

What is a Custom Tool?

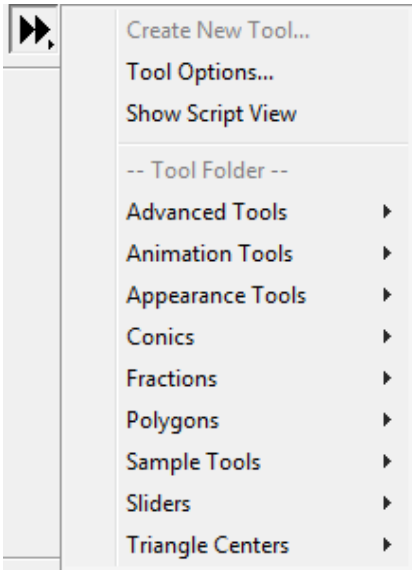
The Geometer's Sketchpad® (GSP) comes with some basic tools, such as the **Straightedge Tool** for drawing lines, rays, and line segments, and the **Compass Tool** for drawing circles based on a centre and a point on the circumference. However, you can create tools to automate the drawing of virtually anything that can be drawn with GSP.

Standard Custom Tools

GSP comes with a number of standard custom tools. However, before you can use them, you must copy them into your **Tool Folder**. The standard custom tools are placed in the **Sketchpad/Samples/Custom Tools** directory when you install GSP on your computer. You must copy the files in this folder to the **Sketchpad/Tool Folder** folder.



Close GSP, and restart. Press and hold the **Custom Tool** button. A list of custom tools appears.



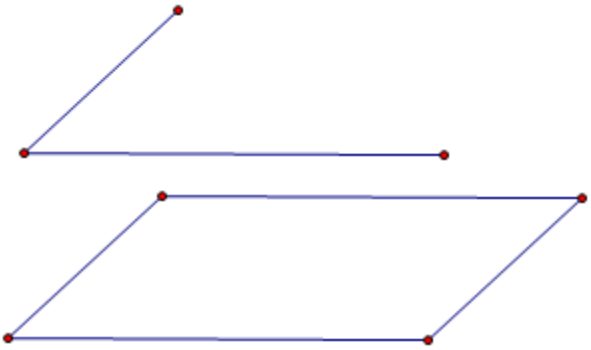
You can experiment with the many different and useful tools available.

Creating a Custom Tool

You can create your own custom tool by making a sketch, selecting the elements that you want to be part of the tool, and then, selecting **Create New Tool...** from the **Custom Tool** menu.

If you save the sketch in your **Tool Folder**, it will appear the next time to start GSP.

Draw a Parallelogram



- Draw two line segments from the same starting point.
- Select one segment, and the endpoint of the other.
- From the **Construct** menu, select **Parallel Line**.
- Repeat to draw another line to complete the figure
- Select the two lines.
- From the **Construct** menu, select **Intersection**.
- Hide the two lines.
- Draw line segments to complete the parallelogram.

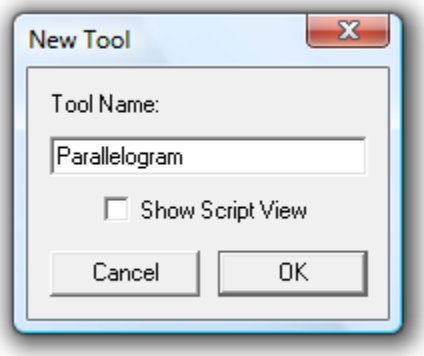
Create a Parallelogram Tool

- Click and drag to form a selection box.

This will select all objects within the box.

- Press the **Custom Tool** button.
- Select **Create New Tool...** .

The **New Tool** box will appear.



- Name the tool **Parallelogram**.
- Press **OK**.

Press the **Custom Tool** button. Notice that the **Parallelogram** tool appears in the list under **This Document**. If you save this document, and give it to someone else, the tool will go with it. However, as soon as you close the document, the tool will disappear from the list.

To save the tool permanently on your computer, you must save the document in the **Tool Folder**. It will then appear the next time you start GSP. Before doing this, we'll create another tool. You can create many tools in one sketch, and then, save them all at once.

Let's draw a few parallelograms before we proceed.

- Press the **Custom Tool** button.
- Select **Parallelogram**.
- Move the pointer to the workspace, and click once.
- Move the pointer, and click again.
- Move the pointer, and click a third time.

Notice that you need to define three points in order to draw a parallelogram.

The parts of the sketch that were hidden, i.e., the two intersecting lines, are not drawn when using the tool. Even if you select **Show All Hidden** from the **Display** menu, you will not see them.

- Select and delete all of the parallelograms except the first one.
- Select the **Text Tool**.
- Type some text such as "This tool draws a parallelogram."

Add a Page

- From the **File** menu, select **Document Options...** .
- Press **Add Page**.
- Select **Blank Page**.
- Press **OK**.

Your new page will appear.

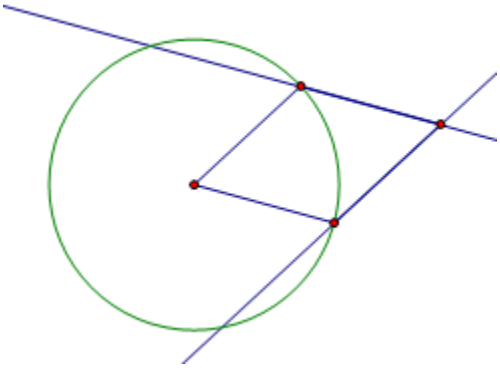
- From the **File** menu, select **Document Options...** .

Change the name of the page to **Rhombus**.

Change the name of the first page to **Parallelogram**. Then, return to the **Rhombus** page.

Draw a Rhombus

- Use the **Compass Tool** to draw a circle.
- Use the **Point Tool** to place another point point on the circle.
- Use the **Straightedge Tool** to draw line segments.
- Construct two lines to form the rhombus.
- Construct the intersection of the lines.



Your sketch will appear as shown.

- Hide the lines and the circle.
- Draw line segments to complete the rhombus.

Create a Rhombus Tool

- Click and drag to form a selection box.
- Press the **Custom Tool** button.
- Select **Create New Tool...** .

The **New Tool** box will appear.

- Name the tool **Rhombus**.
- Press **OK**.

Press the **Custom Tool** button. Notice that both the **Parallelogram** tool and the **Rhombus** tool appear in the list under **This Document**.

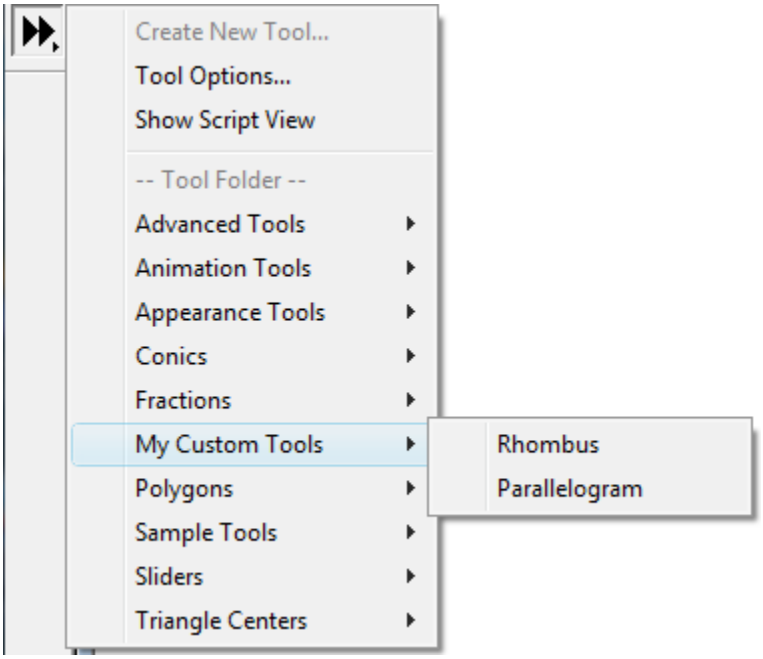
- Press the **Custom Tool** button.
- Select **Rhombus**.
- Move the pointer to the workspace, and click once.
- Move the pointer, and click again.

Notice that you only need to define two points in order to draw a rhombus.

Save Your Tools for Other Sketches

- From the **File** menu, select **Save As...** .
- Navigate to the directory **Sketchpad/Tool Folder**.
- Type a name for the file, such as **My Custom Tools**.
- Press **Save**.

If you press the **Custom Tool** button, you'll notice that the new tools do not appear. You must close and restart GSP. Then, the new tools will appear.



Use *The Geometer's Sketchpad*® to Illustrate a Geometrical Proposition

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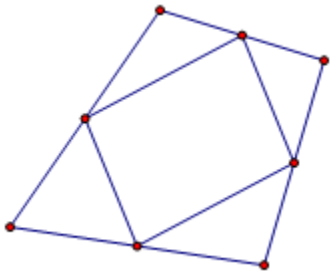
In this activity, you will learn how to use *The Geometer's Sketchpad*® to derive and illustrate a geometrical conjecture.

Part 1: Start with any quadrilateral. Find the midpoints of the sides. Join the midpoints with line segments. Form a conjecture about the figure made up of the segments, and show evidence that your conjecture is correct.



Sketch the Figure

- Use the **Straightedge Tool** to sketch any quadrilateral.
- Select the sides, and construct midpoints.
- Select the midpoints and construct line segments.

The interior quadrilateral appears to be a parallelogram.

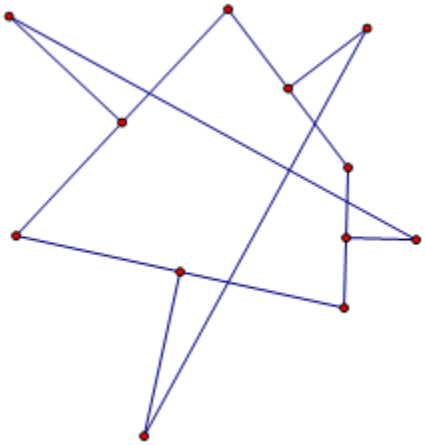
- Select the sides of the interior quadrilateral.
- From the **Measure** menu, select **Slope**.

The slopes of opposite sides are equal. The figure is a parallelogram.

Drag the vertices of the original quadrilateral. The interior figure remains a parallelogram.

Drag any vertex until it overlaps the vertex next to it. Try both directions.

Part 2: Start with any quadrilateral. Find the midpoints of the sides. Erect an external perpendicular at the midpoint, whose length is half the length of the corresponding side. Join the ends of opposite perpendiculars to construct two line segments. Form a conjecture about the two segments, and show evidence that your conjecture is correct.



- Use the **Straightedge Tool** to sketch any quadrilateral.
- Select the sides, and construct midpoints.
- Double-click a midpoint to select it as a centre of rotation.
- Click the point at one end of the same side.
- From the **Transform** menu, select **Rotate...** .
- If necessary, change the angle to rotate outside the quadrilateral.
- Press **Rotate**.
- Repeat for the other three sides.
- Draw line segments for the four perpendiculars.
- Join the ends of opposite perpendiculars.

The two segments appear to be orthogonal.

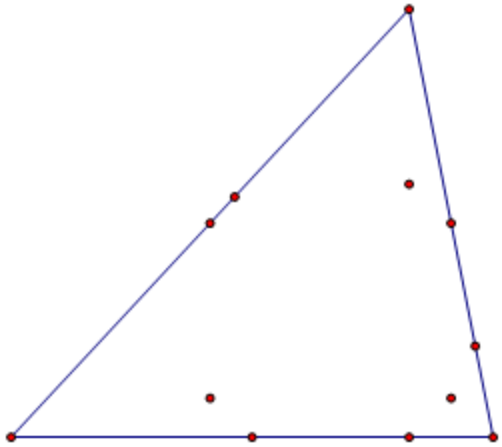
- Construct the intersection of the two segments.
- Measure the angle at the intersection.

The two segments are orthogonal. Drag the vertices of the original quadrilateral. The two segments remain orthogonal.

- Select the line.
- From the **Measure** menu, select **Slope**.

Part 3:

- Use the **Straightedge Tool** to sketch any acute triangle.
- Construct the midpoints of the sides.
- Construct altitudes.
- Construct the orthocentre: where the altitudes meet.
- Construct a point at the foot of each altitude.
- Hide the altitudes.
- Construct a line segment from each vertex to the orthocentre.
- Construct the midpoints of the line segments.
- Hide the orthocentre and the line segments.



You should have nine points visible, other than the vertices of the triangle. These points appear to lie on a circle.

- Make constructions necessary to find the centre of such a circle.
- Hide all constructions except the centre.
- Draw the circle.

Note that it passes through all nine points. This is called the Nine Point Circle.

Drag a vertex of the triangle. Note that the nine points remain on a circle.

Try another sketch, but use an oblique triangle.

Use *The Geometer's Sketchpad*® to Dynamically Display Derivatives

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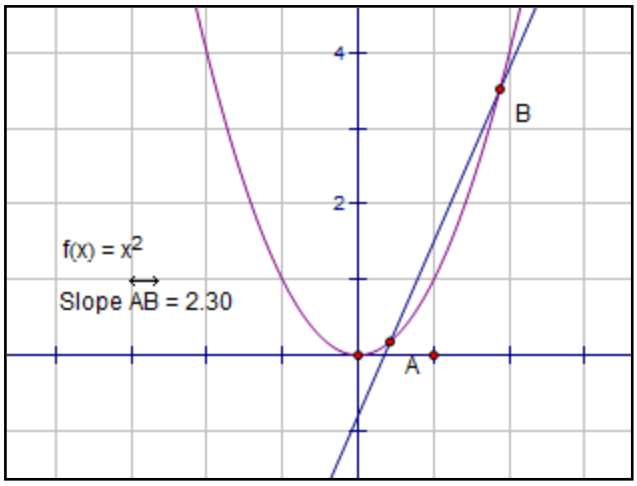
In this activity, you will learn how to dynamically work with derivatives in *The Geometer's Sketchpad*®.

Graph a Function and Draw a Secant

- From the **Edit** menu, select **Preferences...** .
- Select the **Text** tab.
- Check the box **For All New Points**.
- Press **OK**.
- From the **Graph** menu, select **Plot New Function...**

.

- Type *x* ^ 2.
- Press **OK**.



The function and graph will be displayed.

- Use the **Point Tool** to draw two points on the function.
- Construct a line through the two points.
- Select the line.
- From the **Measure** menu, select **Slope**.

Drag each point independently, and observe how the slope of the secant changes.

Calculate the Derivative

- Select the function.
- From the **Graph** menu, select **Derivative**.

The derivative of the function will be displayed.

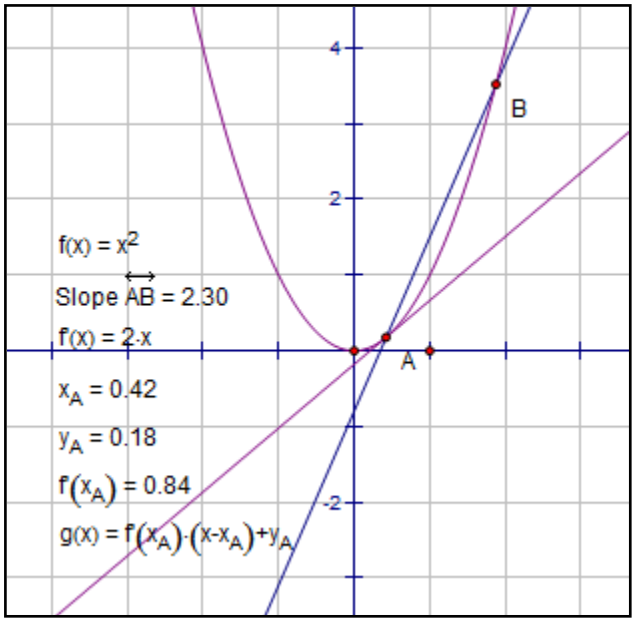
- Measure the coordinates of the first point A.
- Calculate the derivate using the *x*-coordinate of A.
- Drag point B towards point A.

Observe the relation between the slope of the secant and the derivative as point B approaches A.

- From the **Graph** menu, select **Plot New Function...**

.

- Enter the expression f'(x_A)*(x - x_A) + y_A
- Press **OK**.



The tangent at point A will be displayed.

Drag point B towards point A. Observe the secant in relation to the tangent.

Construct Hide/Show Buttons

- Select point B, the secant, and the slope of the secant.
- From the **Edit** menu, select **Action Buttons**.
- Select **Hide/Show**.

A button will be displayed.

- Right-click on the button, and change the label to **Hide Secant**.
- Use the button to hide or show the secant.

In a similar manner, create another button to hide/show the tangent and its measurements.

Hide the secant. Drag point A along the curve, and observe what happens.

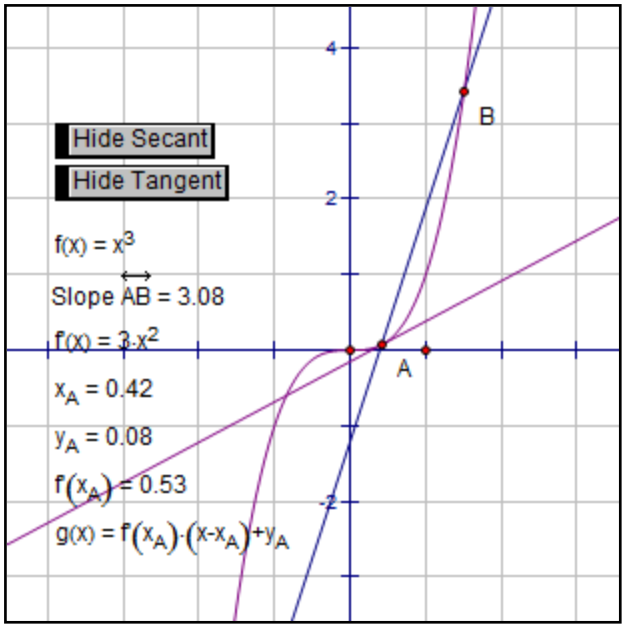
Show the secant.

Try Another Function

- Right-click on the function definition.
- Select **Edit Function...** .
- Change the function to x^3.

Observe what changes have occurred. Experiment with the secant and the tangent.

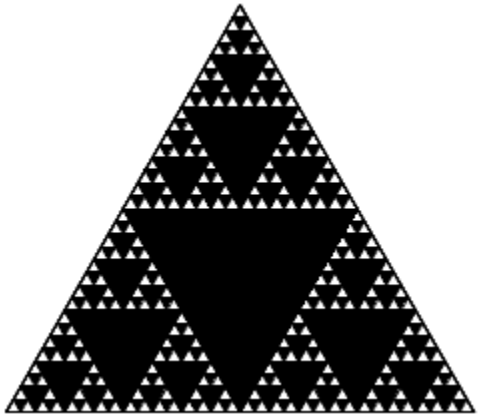
Try other functions of interest.



Use *The Geometer's Sketchpad*® to Create a Sierpinski Triangle: Iteration

(c) 2010-01-31 R. Meisel

In this activity, you will learn how to use iteration in *The Geometer's Sketchpad*® to create a Sierpinski Triangle.



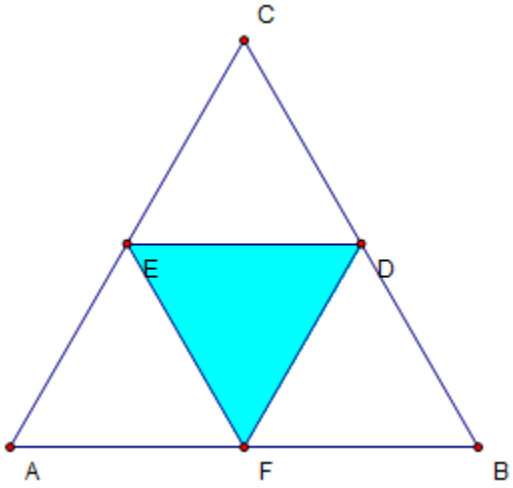
What is a Sierpinski Triangle?

The Sierpinski Triangle is a fractal pattern first described in 1915 by the Polish mathematician Waclaw Sierpinski. The pattern continues indefinitely, and looks the same under any level of magnification. A Sierpinski Triangle with five levels is shown.

Set Up the Triangle

- From the **Edit** menu, select **Preferences...** .
- Select the **Text** tab.
- Check the box **For All New Points**.
- Press **OK**.

This will label all points as you create them.



- Use the **Straightedge Tool** to draw a triangle.
- Select the sides AB, BC, and CA, in order.
- From the **Construct** menu, select **Midpoints**.
- Use the **Straightedge Tool** to join the midpoints.
- Select the three midpoints.
- From the **Construct** menu, select **Triangle Interior**.
- If desired, you can right-click on the triangle, and change the **Color**.

Iterate the Pattern

Map the pattern in the large triangle into each of the three white triangles.

- Select, in order, the points A, B, and C.
- From the **Transform** menu, select **Iterate...** .
- Click on F, B, and D, in order.
- Press the **Structure** button.
- Select **Add New Map**.

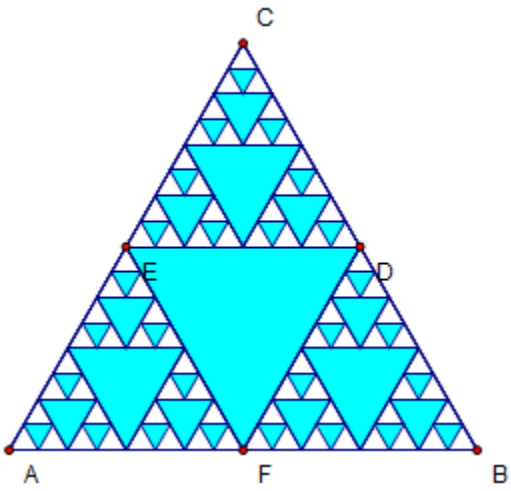
This maps the vertices of the triangle ABC to the vertices of the triangle FBD.

In a similar manner, map ABC onto EDC, and AFE.

- Click on E, D, and C, in order.
- Press the **Structure** button.

- Select **Add New Map**.
- Click on A, F, and E, in order.
- Press **Iterate**.

The mapping will be iterated 3 times, the default setting.



Clean up the diagram by selecting and hiding all of the points.

Change the Iterations

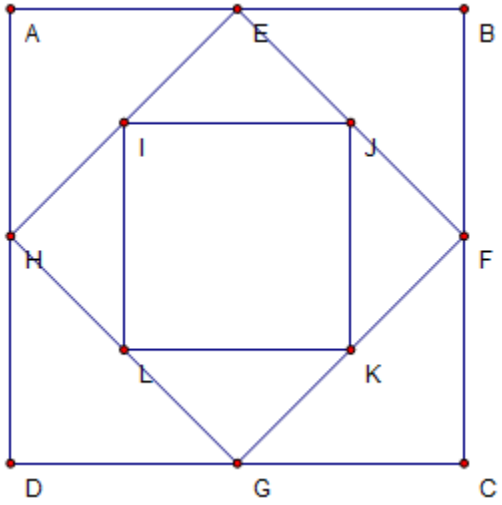
- Use the **Selection Arrow Tool** to draw a selection box around the entire triangle.
- Press + to increase the number of iterations, or – to decrease the number of iterations.

Sierpinski Tile

Extend the method to form a tile.

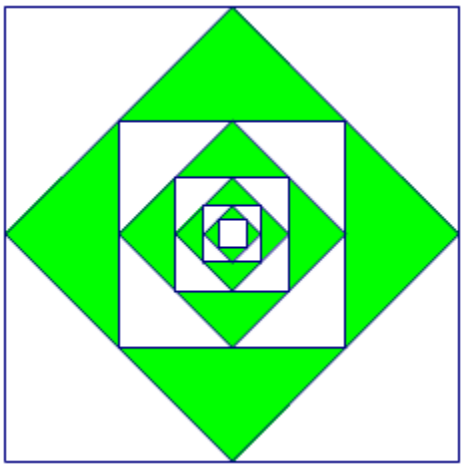
- Draw a square ABCD.
- Construct midpoints for the sides.
- Join the midpoints with line segments.
- Construct midpoints for the segments.
- Join the second set of midpoints.

Your diagram should look much like this at this stage.



- Construct triangle interiors for the smaller triangles.

Determine a mapping that results in the pattern shown. Then, apply and iterate the mapping.



Select the tile, and experiment with the number of iterations.

Note: GSP will differentiate functions much like a Computer Algebra System (CAS).

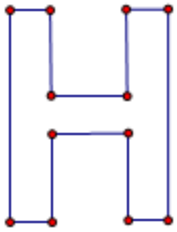
Use *The Geometer's Sketchpad*® for Computer Graphics Imagery (CGI)

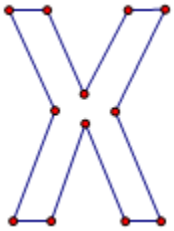
(c) 2010-02-01 R. Meisel

In this activity, you will learn how to use *The Geometer's Sketchpad*® as a CGI program, investigating how computers are used in the motion picture industry to create special effects needed in modern film-making. This kind of software makes heavy use of trigonometry to render three-dimensional objects on a two-dimensional computer screen.

Although *The Geometer's Sketchpad*® is a two-dimensional program, it can be used to represent and animate three-dimensional representations in a limited way.

Part 1: Morph one shape into another.





Create two shapes.

- Use the **Straightedge Tool** to draw a letter H.
- Select all points and line segments, and copy.
- Paste a copy of H, and move it away from the original.
- Draw points on the copy to form the letter X.
- Select all points and line segments on the X.
- From the **Edit** menu, select **Action Buttons**.
- Create a **Hide/Show** button for the X.

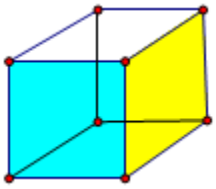
Morph the H into the X.

- Show the X.
- Select the upper left point on the H.
- Select the upper left point on the X.
- Working clockwise, continue selecting points, alternating between H and X.
- From the **Edit** menu, select **Action Buttons**.
- Select **Movement...** .
- Click on the **Label** tab, and change the name to : H to X.
- Press **OK**.
- Hide X.
- Press the H to X button.

The H will morph smoothly into the X. To return the H to the starting position, press **CTRL Z**.

Right-click on the H to X button. Experiment with the movement settings.

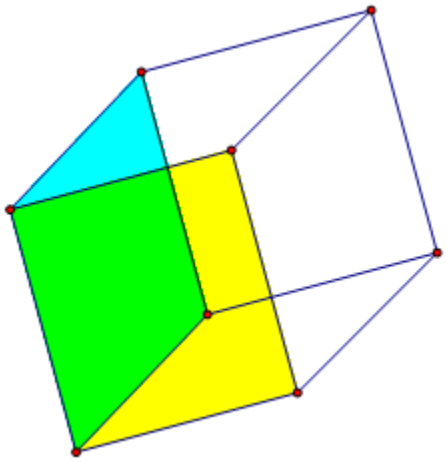
Part 2: Make an Object Fly and Turn



Create starting and finishing shapes.

- Use the **Straightedge Tool** to draw an object cube.
- Create quadrilateral interiors to colour two of the faces as shown.

- Draw a larger, rotated version of the cube to the right and down.
- Determine how points on the object cube match with the image cube.
- Select pairs of matching points, in order.
- From the **Edit** menu, select **Action Buttons**.
- Select **Movement...** .
- Click on the **Label** tab, and change the name to : Fly and Turn.
- Press **OK**.
- Select all elements of the image cube.
- From the **Edit** menu, select **Action Buttons**.
- Create a **Hide/Show** button for the image cube.
- Hide the image cube.
- Press the Fly and Turn button.



The object cube will appear to fly, turn, and move closer.Note that the aspect of the cube does not remain quite right as it flies. To fix this problem, points are usually constrained to fly along a predetermined path. You can`t have spaceships or dinosaurs changing aspect as they move.

Part 3: Show Actions in Sequence

You can combine several motions to form a sequence of actions.

- Draw 8 points close together.
- Create a **Hide/Show** button for the points.
- Map the points on the object cube to the cluster of 8 points.
- Create a movement button. Label it Hyperspace.
- Hide the point cluster.
- Press the Fly and Turn Button, and then the Hyperspace button.

The object cube first flies to the image cube position, and then, off into hyperspace.

Press **CTRL Z** twice to return the object cube to its initial position.

You can create a button that combines the two motions.

- Select the Fly and Turn button, and the Hyperspace button, in order.
- From the **Edit** menu, select **Action Buttons**.
- Select **Presentation...** .
- Select the **Sequentially** button.
- Press **OK**.

A new button called **Sequence 2 Actions** will be displayed. Press this button. The actions will occur sequentially.

Finally, you can made the object disappear after going into Hyperspace. Create a Hide/Show button for the object cube. Select the Fly and Turn, Hyperspace, and Hide Object buttons. Create a presentation button. When you press this button, the object cube will disappear after going into hyperspace.